Third Semester B.Tech. Degree Examination, December 2015
(2008 Scheme)
08.301 : ENGINEERING MATHEMATICS II (CMPUNERFTAHBS)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Evaluate $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$.

2. Find the area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

3. Find the workdone by the force $\overline{F} = x\hat{i} - z\hat{j} - 2y\hat{k}$ in the displacement along the parabola $y = 2x^2$, $z = 2$ from (0, 0, 2) to (1, 2, 2).

4. Obtain the half range sine series for e^x in $0 < x < 1$.

5. Expand $f(x) = x^3$ in $-\pi < x < \pi$ in a Fourier Series.

6. Find the Fourier Cosine transform of e^{-5x}.

7. Form the partial differential equation of all spheres of given radius a and whose centres lie on the xy – plane.

8. Solve $z^2(p^2 + q^2 + 1) = C^2$.

9. Find the particular Integral of $(D^4 - D'^4)z = e^{x+y}$.

10. How many Initial and boundary conditions are required to solve one-dimensional heat flow equation ? In steady state conditions derive the solution of one-dimensional heat equation.

P.T.O.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) Change the order of integration in the integral $I = \int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx$ and evaluate it.

b) Find the volume bounded by the paraboloid $x^2 + y^2 = az$, the cylinder $x^2 + y^2 = 2ay$ and the plane $z = 0$.

c) Evaluate $\int_C (\cos x \sin y - xy) dx + \sin x \cos y \, dy$ by Green's Theorem where C is the circle $x^2 + y^2 = 1$.

12. a) Evaluate $\iint_R x^2 \, dy \, dx$ where R is the two dimensional region bounded by the curves $y = x$ and $y = x^2$.

b) Verify divergence theorem for $\overline{F} = x^2 i + zj + yzk$ taken over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$.

c) Use Stoke's Theorem to evaluate $\int_C yz \, dx + zx \, dy + xy \, dz$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

Module – II

13. a) Find the Fourier series expansion of the periodic function of period 2π, $f(x) = x^2$ in $-\pi < x < \pi$. Hence deduce that

i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$

ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots = \frac{\pi^2}{12}$

iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

b) Find the half range sine series for

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1. \end{cases}$$

c) Find the Fourier transform of

$$f(x) = \begin{cases} x, & |x| < a \\ 0, & |x| > a. \end{cases}$$

14. a) Using Fourier integral representation show that

$$\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda = \begin{cases} \frac{\pi}{2}, & 0 \le x < 1 \\ \pi/4, & \text{for } x = 1 \\ 0, & x > 1 \end{cases}.$$

b) If $f(x) = x - x^2$ in $-\pi < x < \pi$. Deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots = \frac{\pi^2}{12}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}.$$

c) Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$.

Module – III

15. a) Solve $p^2 - q^2 = x - y$.

b) Solve $(D^3 + D^2D' - DD'^2 - D'^3)\,z = e^x \cos 2y$.

c) Write the most general solution of the string equation, if the string (length l) is fixed at both ends and is subjected to zero initial displacement and non zero initial velocity.

16. a) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

b) Solve $z^4p^2 - z^2q = 0$.

c) The ends A and B of a rod 20 cm long have the temperature at 30°C and 80°C until steady. State conditions prevail. The temperature of the ends are changed to 40°C and 60°C respectively. Find the temperature distribution in the rod.
